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ESTIMATION OF WATER QUALITY CRITERIA VIOLATION FREQUENCIES USING PEARSON PERCENTILES

by

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A numerical technique is developed for estimating water quality violation frequencies due to pollutant discharges from urban areas during combined sewer overflow events. The first four moments of in-stream pollutant concentration are found by integrating a pollutant loading - water quality impact model over the probability density functions of its stochastic rainfall and streamflow inputs. The Pearson family of probability distributions is then used to estimate violation frequencies of water quality criteria. This approach proves to be a flexible and efficient method for the initial screening of alternative combined sewer overflow abatement measures.

INTRODUCTION

Measures of receiving water quality are inherently random variables because of the many stochastic influences to which they are subject. These include fluctuations in upstream hydrology and water quality, area-wide stormwater runoff, and treatment plant performance. The effectiveness of controls on both point and non-point pollution sources can be quantified by the reduction in the frequency with which specific water quality criteria, relevant to the intended beneficial use of the water body, are violated. In this way cost-effective levels of pollution reduction can be identified.

Two distinct approaches have been used for predictive modeling of the frequency distributions of water quality variables. Continuous simulation, as represented by SWMM Level III [Medina, 1979] or CSPSS [Wycoff and Mora, 1979] allows use of a high level of model detail at the expense of large input data requirements and computational resources. The derived distribution approach is more suited for use during the early stages of problem assessment and screening of abatement alternatives. It uses simple pollutant loading - water quality response models and statistical assumptions to estimate the distribution of water quality levels as functions of stochastic inputs. The work of Warn and Brew [1980], Loganathan and Delleur [1982], and DiToro [1982] serve as examples.

This paper illustrates how a form of the derived distribution approach can be applied to estimating water quality criteria violation frequencies due to pollutant discharge from an urban area during combined sewer overflow events. It extends previous work in this area by: (a) considering the loading contributions from dry weather sources along with both separate and combined sewer runoff, (b) incorporating both storage and treatment of combined sewer overflows as abatement alternatives, and (c) allowing flexibility in choosing

probability distributions with which to model the stochastic inputs and water quality response output. Although the focus of this work is on combined sewer overflow problems, the approach employed here is general enough to be applied to other types of water quality modeling studies.

CONCEPTUAL FRAMEWORK

Figure 1 is a schematic representation of the urban system being studied. During a storm event the following processes take place: (a) part of the pollutant laden urban runoff enters separate storm drains and is eventually discharged to the receiving water, (b) the remainder of the runoff enters the combined sewer system where it mixes with sanitary sewage, (c) a portion of the combined sewer flow is intercepted and treated at the sewage treatment plant, (d) the remainder becomes combined sewer overflow (CSO) which can be routed through a CSO storage facility, (e) the CSO by-passed from storage can receive special treatment before discharge to the receiving water.

The stochastic inputs to this system are taken to be the interval between storm events, the intensity and duration of storm events, and the available receiving water streamflow. It is possible to include additional probabilistic elements as well, such as treatment plant performance and background in-stream water quality conditions, at the expense of greater computational complexity. We seek to develop a computationally efficient scheme to predict the probability distribution of in-stream water quality conditions during CSO events based on knowledge of: (a) the probability distributions of the stochastic inputs, (b) the runoff quantity and quality characteristics of the urban area, and (c) the available capacities and treatment efficiencies of the sewage treatment plant, CSO storage, and CSO treatment facilities.

The approach to be used is based on a general methodology discussed in

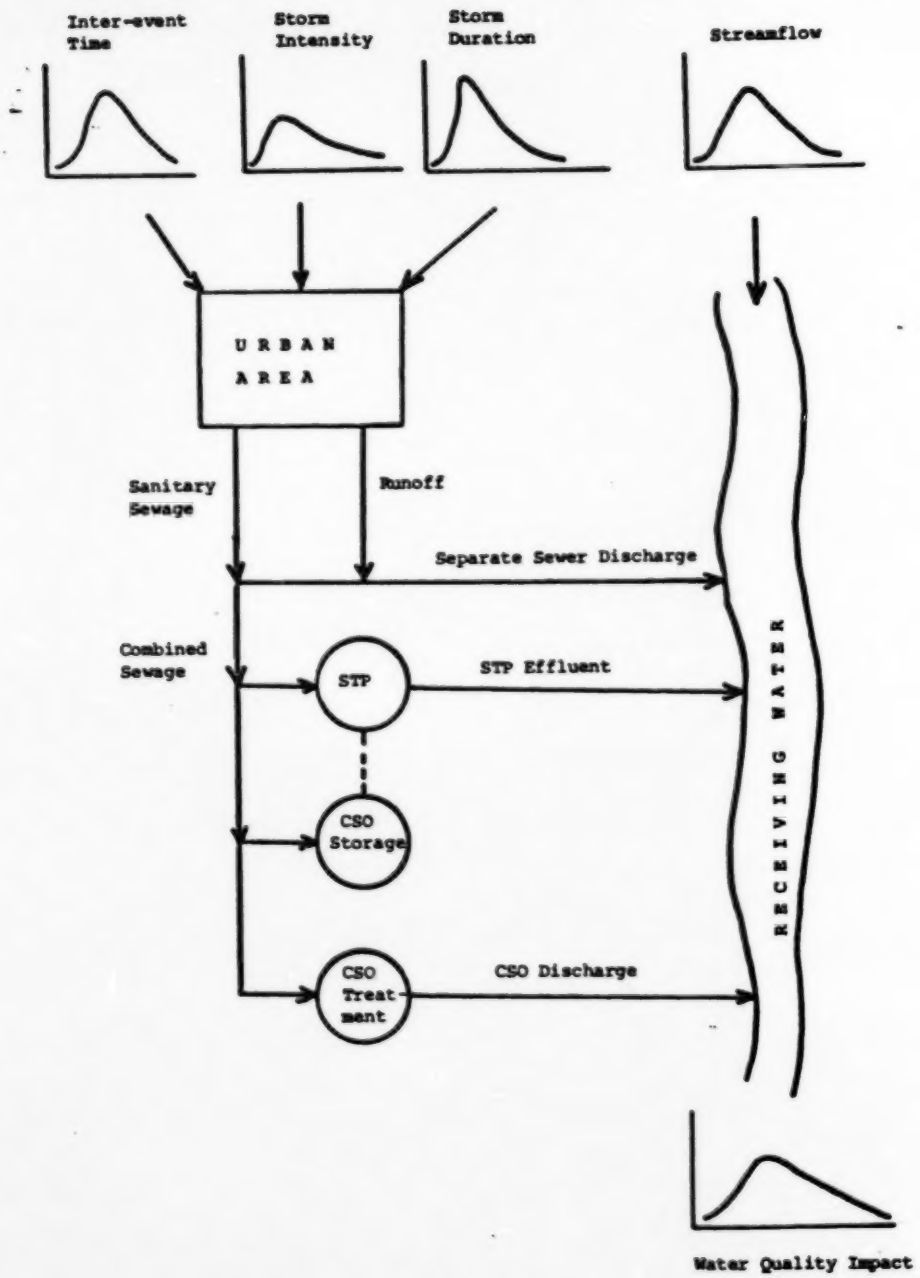

FIGURE 1. Schematic Representation of an Urban Area

Kottas and Lau [1982] and consists of the following steps:

1. Simple runoff and mass balance models are used to derive event-averaged pollutant loadings from all sources as functions of the rainfall characteristics.
2. In-stream mixing with available streamflow and subsequent transport/transformation modeling converts the event-averaged loadings into a resultant in-stream concentration level.
3. The first four central moments of in-stream concentration are found by integrating the runoff-receiving water model with respect to the probability density functions of the stochastic rainfall and streamflow variables.
4. Selected percentiles of the cumulative distribution of in-stream concentration are found from tables of the Pearson family of probability distributions. These can be interpolated to find the probability of violating a specific concentration criterion.

RUNOFF-RECEIVING WATER MODEL

The following lumped, event-averaged runoff model is used to estimate the volume of runoff, v_r, generated by a storm event [Arnell, 1982]

$$v_r = \phi A(id - S_d) \quad \text{.....(1)}$$

where ϕ is a runoff coefficient, A is the area contributing to runoff, i is the event-averaged rainfall intensity (L/T), d is the event duration, and S_d is the depression storage (L).

The mass of a pollutant of interest contained in an event's runoff, m_r,

is represented by the following exponential build-up and washoff model [Alley, and Smith, 1981]

$$m_r = A(K_1/K_2)[1-\exp(-K_2t)][1-\exp(-K_3v_r)] \quad \text{.....(2)}$$

where K_1 is the pollutant accumulation rate constant ($M/L^2/T$), K_2 is the decay rate constant ($1/T$), K_3 is the washoff rate constant ($1/L$), and t is the time between rainfall events.

A CSO event occurs whenever the volume of combined sewer flow exceeds the available sewage treatment plant capacity and available CSO storage facility capacity. The volume of overflow, v_{cso}, can be written as

$$v_{cso} = \alpha\phi A(id - S_d) - \Omega d - s_a \quad \text{.....(3)}$$

where α is the fraction of the urban area served by combined sewers, Ω is the excess hydraulic capacity of the sewage treatment plant (L^3/T), and s_a is the available CSO storage capacity at the start of the event (L^3).

The initial in-stream concentration of a pollutant of interest from a CSO event is found by mixing together all urban loadings with the available volume of receiving water. Denoting this concentration as c_o and applying flow and mass balances around the urban system components shown in Figure 1 results in

$$c_o = g(q,i,d) + h(q,i,d)[1-\exp(-K_2t)] \quad \text{.....(4)}$$

where

$$g = [qC_u + M_{STP} + r(1-\theta)M_{DW}]d/v \quad \text{.....(5)}$$

$$h = [(1-\alpha) + \alpha r(1-\theta)]A(K_1/K_2)[1-\exp(-K_3id)]/v \quad \text{.....(6)}$$

$$v = qd + Q_{DW}d + \phi A(id-S_d) - s_a \quad \text{.....(7)}$$

$$r = (\alpha\phi A(id-S_d) - \Omega d - s_a)/(\alpha\phi A(id-S_d) + Q_{DW}d) \quad \text{.....(8)}$$

and q is upstream streamflow, C_u the upstream pollutant concentration, Q_{DW} the average dry weather flow processed by the sewage treatment plant, M_{DW} the mass flow rate of pollutant in the raw sanitary sewage, M_{STP} the mass flow rate of pollutant discharged by the sewage treatment plant, and θ is the pollutant removal efficiency achieved by CSO treatment which may be flowrate dependent.

Finally, c_o may be modified to some final downstream concentration, c, by means of a transport/transformation function ψ which is assumed to operate linearly on c_o as follows

$$c = c_o \psi \quad \text{.....(9)}$$

where in general, ψ may depend on the average streamflow during the event, v/d.

MOMENTS OF IN-STREAM CONCENTRATION

The final in-stream concentration of a pollutant discharged from a CSO event has been expressed in terms of urban area characteristics which remain constant from event to event (all upper case or Greek characters) and four stochastic variables (q, i, d, and t) which vary statistically from event to event. Therefore c will also vary statistically, in a manner that can be characterized by its moments about the mean.

First, let u_{mo} denote the m-th moment of c about the origin. It can be found by evaluating the integral

$$u_{mo} = \int_{q,i,d,t} c(q,i,d,t)^m \, f'(q,i,d,t|\text{CSO event}) \, dt \, dd \, di \, dq \quad \text{.....(10)}$$

where f' is the joint probability density function of q, i, d, and t

conditioned on the occurrence of a CSO event. A CSO event occurs for those values of i and d that make v_{cso} positive in (3) with duration longer than some specified minimum. Denote this region of i-d space as R. Now we have

$$u_{mo} = \frac{\int_{q,(i,d)\epsilon R,t} c(q,i,d,t)^m f(q,i,d,t)\, dt\, dd\, di\, dq}{\int_{i,d\epsilon R} f(i,d)\, dd\, di} \qquad \text{.....(11)}$$

where f( ) is the marginal joint density function of the variables within the parentheses. The denominator of the above expression yields the probability that a storm event results in a CSO. This term will be denoted as p_{cso}.

With a specific choice of probability density functions for q, i, d, and t the expression for u_{mo} can be evaluated numerically. The Appendix shows how a combination of analytical integration, Gauss-Hermite, and Gauss-Laguerre quadrature can be employed when q is lognormal, i is gamma, and d and t are exponentially distributed and independent of one another. After computing the moments of c about the origin, the moments about the mean, u_m, can be found as follows

$$u_1 = u_{1o}$$

$$u_2 = u_{2o} - (u_{1o})^2$$

$$u_3 = u_{3o} - 3u_{2o}u_{1o} + 2(u_{1o})^3$$

$$u_4 = u_{4o} - 4u_{3o}u_{1o} + 6u_{2o}(u_{1o})^2 - 3(u_{1o})^4$$

PERCENTILES OF IN-STREAM CONCENTRATION

The Pearson system of distribution functions allows a wide variety of shapes in approximating the probability density function of c based on its

first four moments. Bowman and Shenton [1979(a),(b)] have produced a computer oriented method of determining standardized Pearson variates, z_p, at the percentiles p = 1, 2.5, 5, 10, 25, 50, 75, 90, 95, 97.5, and 99 percent in terms of the skewness coefficient $\sqrt{\beta_1} = u_3/(u_2)^{1.5}$ and kurtosis coefficient $\beta_2 = u_4/(u_2)^2$. Thus c_p, the p-th percentile of c (i.e., the concentration not exceeded in more than 100-p % of CSO events) can be computed as

$$c_p = u_1 + z_p\sqrt{u_2} \quad \text{.....(12)}$$

Let $p_{v|cso}$ be the probability of violating a water quality criterion c_{std} during a CSO event. It can be found by interpolating for the p that corresponds to $z_p = (c_{std}-u_1)/\sqrt{u_2}$ among the percentiles listed above and subtracting it from 100. Then the average number of water quality violations per year due to CSO's, n_v, is

$$n_v = p_{v|cso}\, p_{cso}\, n_e \quad \text{.....(13)}$$

where n_e is the number of storm events per year. The reduction achieved in n_v is one possible measure of effectiveness to use in comparing alternative levels of CSO abatement.

One limitation of the Pearson system of distributions is that percentiles have only been tabulated for skewness coefficients of 2 or less in absolute value. For positive skewness greater than two it may be possible to employ the lognormal distribution instead without incurring excessive error. In this case the p-th percentile of c would be $\exp(\mu + z_p\sigma)$ where $\mu = \ln(u_1) - .5\sigma^2$ and $\sigma^2 = \ln(1 + u_2/u_1^2)$. Now z_p would be found from a table of standard normal variates or its functional approximation [Abramowitz and Stegun, 1964].

Likewise, $p_{v|cso}$ could be estimated by finding the probability that corresponds to the standard normal variate $z_p = [\ln(c_{std}) - \mu]/\sigma$ and subtracting it from 100.

ILLUSTRATIVE EXAMPLE

The data listed in Table 1 are meant to be representative of conditions during a four month recreational season for a small community discharging to a main drainage river. It will be used to estimate how the distribution of coliform bacteria two miles downstream of the urban area would be impacted by CSO's and their storage or treatment.

For this example we assume that the in-stream coliform transport/transformation function is

$$\psi = \exp[-10.56(\text{streamflow, cfs})^{-.4}] \quad \text{.....(14)}$$

which is based on a first order coliform die-off with rate coefficient of 0.36 1/hr. The efficiency of CSO disinfection with chlorine is assumed to be given by [Driscoll, et al., 1979]

$$\theta = 1 - (1 + .3c_L\tau)^{-3} \quad \text{.....(15)}$$

where c_L is the residual chlorine dose applied in mg/L and τ is the contact time in minutes. Three different levels of CSO abatement will be examined. Case 1 represents the uncontrolled condition, Case 2 provides 11.3 mil gal of CSO storage, while Case 3 employs a CSO disinfection system with a chlorine feed capacity of 50 lb/day and a 12,500 gal contact basin.

A small computer program was written to implement the moments integration

TABLE 1. Sample Urban Area Characteristics

Drainage area, sq mi	2.6
Fraction served by combined sewers	0.8
Average dry weather sanitary flow, mgd	2.0
Maximum sewage treatment plant capacity, mgd	3.0
Coliform concentration in raw sewage, 10^6 MPN/100 mL	1.0
Coliform concentration in treated sewage, MPN/100 mL	200.0
Mean time between storms, hr	60.0
Mean storm duration, hr	7.0
Mean storm intensity, in/hr	0.025
Std. deviation of storm intensity, in/hr	0.0275
Depression storage, in	0.02
Runoff Coefficient	0.38
Dry weather coliform accumulation rate, 10^9 MPN/acre/day	1.26
Dry weather coliform decay rate, 1/day	0.1
Wet weather coliform washoff rate, 1/in	4.0
Mean upstream receiving water streamflow, cfs	1240.0
Std. deviation of upstream streamflow, cfs	1488.0
Mean upstream coliform concentration, MPN/100 mL	0.0
In-stream coliform die-off rate, 1/hr	0.36

scheme described in the Appendix using 5 point Hermite and 10 point Laguerre quadrature. It also computes the 11 Pearson percentiles of in-stream concentration listed above. Figure 2 shows the resulting distribution of coliform concentrations at the stream impact point caused by CSO events longer than two hour duration for cases 1, 2, and 3.

Now suppose that a 400 MPN/100 mL coliform criterion was in effect. Based on 43 storm events per 4 month recreational season, the number of coliform violations expected each season would be as follows

	Case 1	Case 2	Case 3
Probability of Violation | CSO event	0.29	0.18	0.09
Probability of CSO Event	0.57	0.27	0.57
Expected Number of Violations per Season	7	2	2

Thus even though treatment has a much more moderating effect on in-stream coliform levels, 11.3 mil gal of storage provides the same reduction in coliform violations because of the reduction of CSO events it achieves.

These three particular cases were chosen merely to illustrate the type of behavior one might expect using the model to evaluate alternative abatement options. Naturally, a much more comprehensive analysis would be required for any actual cost-effectiveness study of CSO control.

CONCLUSIONS

The urban runoff - receiving model used in this study offers an efficient tool for preliminary analysis of CSO water quality impacts. Its computerized version can be applied in an interactive fashion to quickly evaluate the effectiveness of alternative CSO storage/treatment combinations in reducing

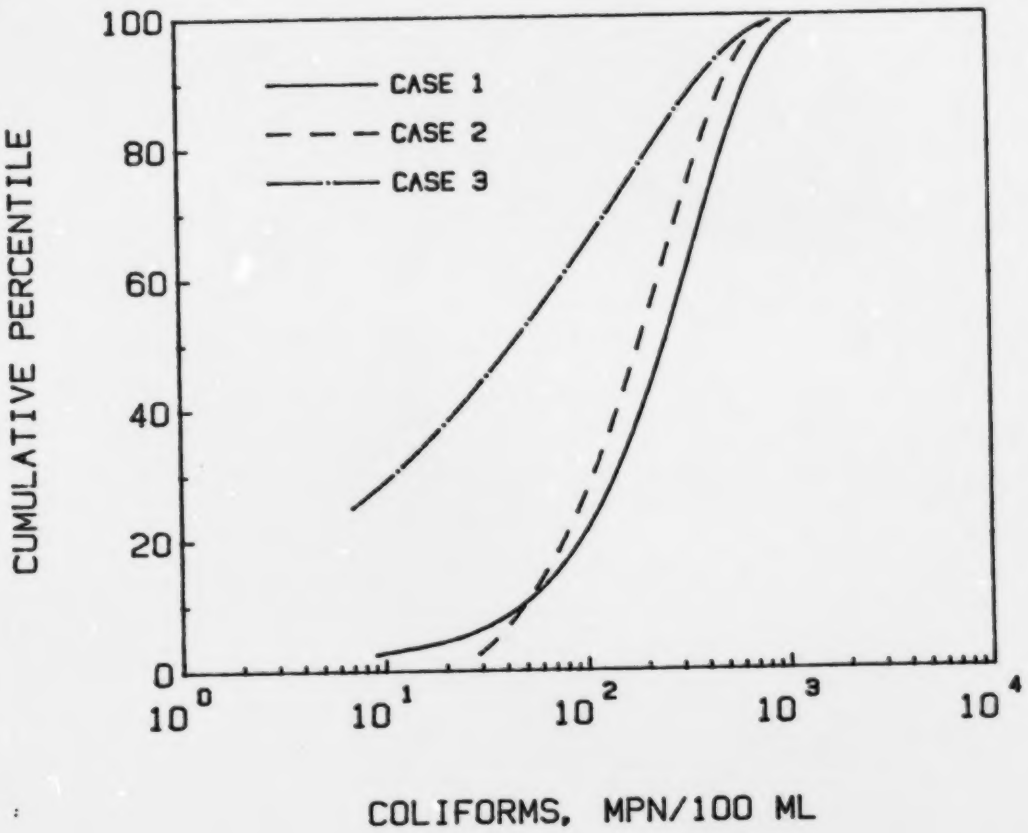

FIGURE 2. Cumulative Distribution of In-Stream Coliforms

water quality violations. It can also serve as a subroutine in a more structured cost-effectiveness model. Such a model would employ a numerical optimization procedure to find the storage/treatment design that meets a designated water quality violation frequency at least total cost.

The four moments - Pearson percentile approach is general enough to be applied to other types of water quality problems besides CSO abatement. Its main advantage is the flexibility it provides to derived distribution modeling. Various types of pollutant loading models, receiving water quality models, and statistical representations of stochastic inputs can be accomodated. The versatility of the Pearson family of distributions insures that this flexibility is also maintained in estimating the frequency distribution of the water quality output being modelled. One disadvantage of the approach is that as more stochastic input variables are included, the numerical integration requirements increase exponentially. In addition, the distributions of highly skewed water quality responses may not fall within the limits of the currently tabulated values of Pearson percentiles.

APPENDIX

Consider the case where the time between storms, t, is independent of the other stochastic variables and is exponentially distributed with mean T. Substitution of Eq. (4) into Eq. (11) and integration with respect to t gives

$$u_{mo} = \frac{\int_q \int_{i,d \epsilon R} c_m^*(q,i,d)\, f(q,i,d)\, dd\, di\, dq}{\int_{i,d \epsilon R} f(i,d)\, dd\, di} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

where $c_m^* = \psi^m \left[(g+h)^m + \sum_{j=1}^{m} (-1)^j \binom{j}{m} h^j (g+h)^{m-j}/(1+jK_2T) \right]$

and g and h are given by Eq. (5) - (8).

Now let q, i, and d be independent of one another and be lognormally, gamma, and exponentially distributed, respectively. Then it is possible to numerically integrate Eq. (16) quite efficiently using Gauss-Hermite quadrature with respect to q and double Gauss-Laguerre quadrature with respect to i and d [Carnahan, et al., 1969]. If N_H Hermite points and N_L Laguerre points are used in these quadratures then u_{mo} can ultimately be expressed as the weighted sum of $N_H N_L^2$ evaluations of c_m^*.

Evaluation of c_m^* requires knowledge of s_a, the available CSO storage at the start of a storm event. As an approximation, it can be replaced by its mean value, $\bar{s}_a$. Using a procedure similar to that of DiToro and Small [1979] we obtain the following

$$\bar{s}_a = \int_{i,d} \{\Omega T[1-\exp(-S-v_e)/\Omega T)]+v_e\} f(i,d)\, dd\, di \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

where $v_e = \max[0 , \bar{s}_a - \max[0 , \alpha\phi A(id-S_d)-\Omega d]]$ and S is the empty storage volume. This nonlinear equation can be solved by a method of successive approximations beginning with $\bar{s}_a = S$ and employing a double Gauss-Laguerre quadrature to numerically evaluate the integral.

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